Probability: Multiple Events (Dependent and Independent)

To find the probability that a 1st event happens ***and*** a 2nd event also happens, ***multiply*** the two probabilities.

I. Dependent Events

When events are dependent, the second probability changes depending on what event happened first.

Rule for Dependent events: If A and B are dependent events, $P(A \text{ and } B) = P(A) \cdot P(B|A)$

1. A refrigerator has 7 apples and 5 pears.

a. Find the probability that the first piece of fruit is an apple and the second piece of fruit is a pear.

$P(A) = \frac{7}{12}$ $\quad P(B|A) = \frac{5}{11}$ → only 11 pieces left after the apple was taken

$\frac{7}{12} \cdot \frac{5}{11} = \frac{35}{132} = .265 = 26.5\%$

b. Find the probability that both pieces of fruit are pears.

$P(1\text{st pear}) = \frac{5}{12}$ $\quad P(2\text{nd pear}) = \frac{4}{11}$ ↳ After 1st pear was taken

$\frac{5}{12} \cdot \frac{4}{11} = \frac{5}{33} = .152 = 15.2\%$

2. There are 10 soda cans in a refrigerator: 3 regular sodas and 7 diet sodas. Suppose that two cans are taken from the refrigerator for drinking, one after another.

a. What is the probability that the two sodas taken are both diet sodas?

$P(1\text{st Diet}) \cdot P(2\text{nd Diet after 1st was taken})$

$\frac{7}{10} \cdot \frac{6}{9} = \frac{7}{15} = .467 = 46.7\%$

b. Write a probability question about the sodas whose answer would be $\frac{3}{10} \cdot \frac{2}{9} = \frac{6}{90}$.

What is the probability that 2 regular sodas are taken from the fridge?

(3 out of 10 are regular. Since 1 is taken from the numerator & denominator of the 2nd probability then we know it is a 2nd reg. soda that was taken)

3. A deck of cards has 52 cards: 13 hearts (H) and 39 non-hearts (N). Two cards are dealt from the deck, one card after the other.

a. What is the probability that the first card is a heart but the second card is not a heart?

$P(\text{heart}) \cdot P(\text{Non-heart after 1 card is removed})$

$\frac{13}{52} \cdot \frac{39}{51} = \frac{13}{68} = .191 = 19.1\%$

b. What is the probability that both cards are hearts?

$P(\text{heart}) \cdot P(\text{heart after 1st heart is removed})$

$\frac{13}{52} \cdot \frac{12}{51} = \frac{1}{17} = .059 = 5.9\%$

Name: Algebra 2
Date:

II. Independent Events

When events are independent, the second probability doesn't change regardless of what happened in the first event. In this case, finding the answer to an "and" probability problem is simpler: you just find the two probabilities separately, then multiply.

Rule for independent events: If A and B are independent events, $P(A \text{ and } B) = P(A) \cdot P(B)$

4. One jar has 5 red marbles and 3 yellow marbles. Another jar has 4 green marbles and 6 blue marbles. Suppose that one marble is randomly drawn from each jar.

8 total 10 total

→ 2 jars so the 2nd prob. is not affected by the 1st

a. What is the probability of getting a red marble and a green marble?

P(Red) · P(green)

$\frac{5}{8} \cdot \frac{4}{10} = \frac{1}{4} = .25 = 25\%$

b. What is the probability of getting a red marble and a blue marble?

P(Red) · P(blue)

$\frac{5}{8} \cdot \frac{6}{10} = \frac{3}{8} = .375 = 37.5\%$

c. Write a question about the marbles whose answer would be $\frac{3}{8} \cdot \frac{6}{10} = \frac{18}{80}$.

What is the probability that a yellow and blue marble is chosen?

5. Suppose you roll two 6-sided dice. One die is red and the other die is green.

a. What is the probability of rolling a "3" on the red die and a "5" on the green die?

P(3) · P(5)

$\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36} = .028 = 2.8\%$

b. What is the probability of rolling a "3" on the green die and a "5" on the red die?

$\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36} = .028 = 2.8\%$

* the color of the die does not affect the probabilities.

c. What is the probability of rolling a "3" and a "5" on the two dice?
Hint: Combine the answers from parts **a** and **b**.

$\frac{1}{36} \cdot \frac{1}{36} = \frac{1}{1296}$

d. What is the probability of rolling a "4" and a "4" on the two dice?

$\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

Name: Algebra 2
Date:

6. Probabilities of getting different color gumballs are given below. Answer these questions about getting two gumballs from a machine. Assume the colors are independent (the color you get on the first gumball does not affect the probabilities for the second gumball).

color	probability
red	0.2
white	0.15
blue	0.1
pink	0.27
orange	0.07
green	0.2
gold	0.01

↳ prob. add to = 1 or 100%

a. What is the probability that the first gumball is green and the second gumball is blue?

$P(\text{green}) \cdot P(\text{blue})$
$.2 \cdot .1 = .02$

b. What is the probability that both gumballs are white?

$P(\text{white}) \cdot P(\text{white})$
$.15 \cdot .15 = .0225$

c. What is the probability that the first gumball is red and the second gumball is **not** red?

$P(\text{red}) \cdot P(\text{NON-Red})$
$.2 \cdot .8 = .16$

d. What is the probability that both gumballs are gold? (If your calculator gives you an answer in "E" notation, write it as an ordinary decimal.)

$P(\text{gold}) \cdot P(\text{gold})$
$.01 \cdot .01 = .0001$

Mixed Problems: dependent and independent

In the following, think carefully about whether the events involved are in each question are **dependent** or **independent**.

7. Paige has 9 pens in her backpack: 6 blue and 3 red. Here are two slightly different questions about the pens (the only difference is highlighted in bold).

a. Paige randomly takes a pen from her backpack to take notes in English. She **puts the pen away** at the end of class. Next period in Social Studies, again she randomly takes a pen from her backpack.

she puts the pen back Independent!

What is the probability that Paige used a red pen in English and a blue pen in Social Studies?

$P(\text{Red}) \cdot P(\text{Blue})$
$\frac{3}{9} \cdot \frac{6}{9} = \frac{2}{9} = .22 = 22\%$

b. Paige randomly takes a pen from her backpack to take notes in English. She **forgets to put that pen away**, leaving it on her English desk. Next period in Social Studies, again she randomly takes a pen from her backpack.

pen is removed! Dependent

What is the probability that Paige used a red pen in English and a blue pen in Social Studies?

$P(\text{Red}) \cdot P(\text{Blue})$
$\frac{3}{9} \cdot \frac{6}{8} = \frac{1}{4} = .25 = 25\%$

Name: Algebra 2
Date:

8. A child's toy box contains 7 rectangle blocks and 5 triangle blocks.

a. Suppose that a block is taken from the toy box, and **not put back**. Dependent!
Then, another block is taken from the toy box.

What is the probability that a triangle block was taken the first time and a rectangle block was taken the second time?

$P(\Delta) \cdot P(\square \text{ after } \Delta \text{ was taken})$

$\frac{5}{12} \cdot \frac{7}{11} = \frac{35}{132} = .265 = 26.5\%$

b. Suppose that a block is taken from the toy box, and then **returned**. Independent
Then, a block is taken from the toy box again.

What is the probability that a triangle block was taken the first time and a rectangle block was taken the second time?

$P(\Delta) \cdot P(\square)$

$\frac{5}{12} \cdot \frac{7}{12} = \frac{35}{144} = .243 = 24.3\%$

9. Here is some given information about events A, B, C, and D:

- $P(A) = 0.4$
- $P(B) = 0.3$
- $P(C) = 0.2$
- If event C occurs, then the probability of D is 0.65
- If event C does not occur, then the probability of D is 0.8

a. Are events C and D independent? Explain why or why not.

The probability changes when C does not occur so they are dependent events

b. Find $P(A \text{ and } B)$.

$P(A) \cdot P(B)$

$.4 \cdot .3 = .12$

c. Find the probability that neither A nor B occurs.

$P(\text{NO } A) \cdot P(\text{NO } B)$

$.6 \cdot .7 = .42$

The prob. that A does occur is .4 or 40%
So the prob. that A does NOT occur is .6 or 60%
Prob. adds to = 1 or 100%

d. Find $P(C \text{ and } D)$.

$P(C) \cdot P(D \text{ if } C \text{ occurs.})$

$.2 \cdot .65 = .13$