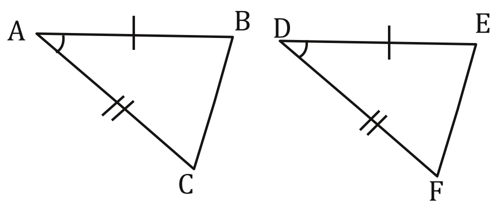
Proofs Involving Congruent Triangles

First, let's analyze some proofs.

This is easy! All you have to do is explain in plain English what is going on in the proofs. We'll look at some examples first.

AE. 1.

Given: $\overline{AB}\cong\overline{DE}$, $\overline{AC}\cong\overline{DF}$, and $\angle A\cong\angle D$



Prove: $\triangle ABC\cong\triangle DEF$

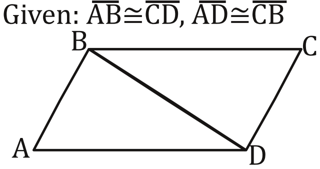
Statements	Reasons
1. $\overline{AB}\cong\overline{DE}$	1. Given
2. $\overline{AC}\cong\overline{DF}$	2. Given
3. $\angle A\cong\angle D$	3. Given
4. $\triangle ABC\cong\triangle DEF$	4. SAS

Analysis:

Working backward we must ask the key question, "How can we show that two triangles are congruent?" The answer? A triangle congruence theorem like SSS, SAS, ASA, AAS or HL. This gives us B1: $\triangle ABC\cong\triangle DEF$, by some property, but which one? To find out, start working forward. Listing all of the given information gives us a pair of angles $\angle A$ and $\angle D$ sandwiched between a pair of congruent sides $\overline{AB}\cong\overline{DE}$ and $\overline{AC}\cong\overline{DF}$. So this means we have $\triangle ABC\cong\triangle DEF$ by the SAS theorem which is B2: and the proof is complete.

AE. 2.

Given: $\overline{AB}\cong\overline{CD}$, $\overline{AD}\cong\overline{CB}$



Prove: $\triangle ABD\cong\triangle BCD$

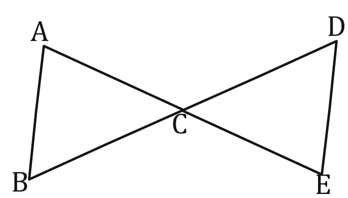
Statements	Reasons
1. $\overline{AB}\cong\overline{CD}$	1. Given
2. $\overline{AD}\cong\overline{CB}$	2. Given
3. $\overline{BD}\cong\overline{BD}$	3. Reflexive property
4. $\triangle ABD\cong\triangle CDB$	4. SSS

Analysis:

Working backward, we must ask the key question "How can we show that two triangles are congruent?" The answer? A triangle congruence theorem like SSS, SAS, ASA, AAS or HL. This gives us B1: $\triangle ABC\cong\triangle BCD$ bys ome property, but which one? Then start working forward. Listing all of the given information gives us two pairs of sides $\overline{AB}\cong\overline{CD}$ and $\overline{AD}\cong\overline{CB}$, but this is not enough. We need another pair of sides or an angle between them. Looking now at the diagram we have $\overline{BD}\cong\overline{BD}$ as a shared line. So this brings us to say $\triangle ABC\cong\triangle BCD$ by SSS which is B1 and the proof is complete.

AE. 3.

Given: $\overline{AE}$ Bisects $\overline{BD}$, $\angle B\cong\angle D$



Prove: $\triangle ABC\cong\triangle DBC$

Statements	Reasons
1. $\angle B\cong\angle D$	1. Given
2. $\overline{AC}$ Bisects $\overline{BD}$	2. Given
3. $\overline{BC}\cong\overline{DC}$	3. Definition of Bisect
4. $\angle ACB\cong\angle DCE$	4. Vertical angles
5. $\triangle ABC\cong\triangle DBC$	5. ASA

Analysis:

Working backward we must ask the key question, "How can we show that two triangles are congruent?" The answer? A triangle congruence theorem like SSS, SAS, ASA, AAS or HL. This gives us B1: $\triangle ABC\cong\triangle BCD$ by some property, but which one? Then start working forward. Listing all of the given information gives us a pair of angles $\angle B$ and $\angle D$, and $\overline{BD}$ and $\overline{AE}$ bisects $\overline{BD}$. If AE bisects $\overline{BD}$ then $\overline{BD}$ is cut in half at C so $\overline{BC}\cong\overline{DC}$! This is not enough though. Looking at the diagram we see vertical angles $\angle ACB\cong\angle DCE$, which gives us $\triangle ABC\cong\triangle BCD$ by the property ASA . This is B1 and the proof is complete.